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RESEARCH MEMORANDUM

INVESTIGATION TO DETERMINE CONTRACTION RATIO FOR SUPERSONIC-COMPRESSOR ROTOR

By Linwood C. Wright

Flight Propulsion Research Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
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SUMMARY

A method of applying the limiting contraction ratio to obtain the maximum allowable blade thickness of a two-dimensional model of a supersonic-compressor rotor is proposed. This method requires the location of the point on the rear of the blade entrance where the tangent line is parallel to the undisturbed relative flow velocity in rotor coordinates. A graphical procedure for approximating this point with what is believed to be sufficient accuracy is presented. For a given blade spacing, the blade-passage area is then supposed to be simply a function of the blade thickness and the sine of the angle between the stagger line and the flow direction. Experimental evidence that this supposition is reasonably accurate is presented along with the suggestion that some additional thickness may be obtained by further turning of the stream in the supersonic region. In addition, the suggestion is made that the two-dimensional characteristic lines be constructed to insure that no subsonic nor too-low supersonic regions result before the minimum section in the passage entrance from reflected compression waves.

INTRODUCTION

A logical result of effort to extend and to improve the performance of axial-flow compressors has been an increase in interest in supersonic axial-flow compressors since 1942. A most promising approach is that first proposed in references 1 to 3. Supersonic-compressor rotor blades essentially comprise a series of small rotating supersonic diffusers whose function is exactly the same as that of conventional supersonic diffusers with internal diffusion. Hence, it has been proposed at the NACA Langley Field laboratory that the blade passages be considered as diffusers and designed on the basis of supersonic-diffuser data (references 1 and 2). Reference 3 reports results obtained on a supersonic compressor run in Freon-12 (a commercial refrigerant). Use of Freon-12 permitted operation with about one-fourth the stress resulting from operation in air at approximately the same Mach number and velocity diagram.

A supersonic-compressor investigation is being conducted at the NACA Cleveland laboratory to prove the feasibility of operating supersonic compressors in air and to continue research on the aerodynamic problems. A compressor rotor with the form of blading described in reference 1, but with larger blades, was designed for operation at approximately twice the rotor speed used in the investigation reported in reference 3. Preliminary blade-vibration checks indicated that the vibrational stresses were so severe that the combined vibrational and centrifugal stresses might exceed the yield point of the blade material. A more accurate examination of the factors governing the contraction ratio and the blade thickness was therefore required.

The contraction-ratio expression is reexamined, this time with consideration of the blade curvature. Results of an experimental investigation made to determine the validity of this analysis are also presented. A cascade of five supersonic blades having the form and spacing for the blade-root (most critical) section of the supersonic compressor rotor blades was constructed and investigated for shock entry in an open-jet supersonic wind tunnel. The blade thickness giving the limiting contraction ratio, which was predicted for the design Mach number by the analysis, and three additional blade thicknesses were used. A simple guide and an experimental substantiation are thus provided for the design of the thicknest practical supersonic-compressor blading of the type used in reference 3.

The assumption that a two-dimensional analysis is permissible is based on the hypothesis that a least approximate simple radial equilibrium can be established before and maintained after the blade-contained shock by control of the initial tangential-velocity distribution and the spanwide distributions of the inlet Mach number and flow angle.

A vibration investigation conducted simultaneously by the Stress and Vibration Section of the Cleveland laboratory will be reported separately.

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SYMBOLS

The following symbols are used in this report:

A	area
a	velocity of sound, feet per second
C_R	contraction ratio, A_e/A_{min}
$C_{R,i}$	isentropic contraction ratio, $(\rho V)_{M=1}/(\rho V)_M$
$C_{R,max}$	limiting nonisentropic compression ratio, $C_{R,i}(P_3/P_0)$
d	diameter to given blade section, inches
M	Mach number
M_0	free-stream Mach number
n	number of blades in complete rotor
P	total pressure, pounds per square foot absolute
t_{max}	maximum blade thickness, inches
V	velocity, feet per second
γ	ratio of specific heat at constant pressure to specific heat at constant volume
ρ	density, slugs per cubic foot
φ	angle between compressor axis and air velocity relative to rotor, degrees

Subscripts:

0	upstream stagnation conditions
1	stagnation condition before shock (relative to rotor)
2	static, or stream, condition (relative to rotor)
3	stagnation condition after shock (relative to rotor)
e	entrance

M at local Mach number

max maximum

min minimum 882

rot rotational

The following point-location letters (fig. 1(b)) may also be used as subscripts:

a point locating blade leading edge

a' point locating blade leading edge adjacent to a

b point locating blade maximum thickness

f point locating blade trailing edge

x origin (on blade convex, or rear, side) of expansion wave that intersects point a'

CASCADE CONTRACTION-RATIO THEORY

The adaptation of contraction ratio, one of the principal parameters governing conventional-supersonic-diffuser performance (reference 1), to rotor blading cannot be accomplished through simple one-dimensional analysis. The use of this important parameter in cascade work was first attempted quantitatively in reference 3. Figure 1(a) shows the contraction ratio as defined by reference 3. For cascades with a straight entrance region (straight rear side of blade up to the minimum section), this interpretation would be accurate. However, for a curved entrance region (convex blade rear side, line axb, fig. 1(b)), the situation is slightly more complex. From figure 1(b), it is obvious that expansion waves are given off along the blade surface if the flow follows the blade contour. When the magnitude of the velocity component normal to the stagger line is less than sonic, the mass flow, the direction of flow, and hence the contraction ratio C_R are fixed by these waves. Consequently, an analysis considering these factors is necessary to compute an accurate value of the contraction ratio.

The method offered determines the entrance and minimum areas used in finding the contraction ratio of a supersonic cascade in terms of blade thickness and flow direction. This method is applied to blades with curved entrance regions and reduces to that shown in reference 3 and figure 1(a) when the blade-entrance region is straight. The method is further restricted to blades of nearly uniform curvature.

Calculation of Minimum Area

The blade-passage minimum area may be computed from the angle $(90^\circ - \varphi)_b$ between the tangent to the line abf (fig. 1(b)) at the maximum-thickness point b and the stagger line, the blade gap, and the maximum blade thickness. The minimum area is given very closely by the expression $(\pi d/n) \sin (90^\circ - \varphi)_b - t_{max}$ for any blade section of mild curvature (fig. 1(a)). This area now corresponds to A_{min} (fig. 1(c)). If any question exists as to the minimum-area location, the computations may be made for several points by using the angle $(90^\circ - \varphi)$ and the thickness at the same point and selecting the smallest value for the minimum area.

Calculation of Entrance Area

In order to calculate the entrance area, it is necessary to find the point x along the entrance region of the blade at which the relative velocity, obtained from the vector addition of the rotational velocity and the undisturbed upstream velocity, is tangent to the blade. This point x determines the direction of the entrance flow and can be found from the curved-blade hypothesis proposed in reference 2. The hypothesis explains that in this type of blading (see reference 3) the flow is tangent, not to the blade leading edge as in a blade of straight entrance region, but to some point x (fig. 1(b)). The blade will thus operate with an oblique shock from the rearward side of its leading edge followed by expansion waves.

When the flow through the wave configuration in front of a supersonic cascade of only slightly curved blades very closely approaches isentropic conditions and the rotor velocity is fixed, the angle of the undisturbed flow may be approximated as follows:

A point x along the surface ab (fig. 1(b)) is selected to which the undisturbed flow is assumed tangent in rotor coordinates. (This selection can be made if the flow is assumed isentropic.) If the flow direction relative to the blades, the rotational velocity, and the direction of the absolute velocity are known, the flow is entirely known from the vector triangle (fig. 1(b)), and the Mach line may be drawn at the point x. If this Mach line does not end at a', another point x must be assumed and the

Mach line drawn as before until the Mach wave that joins x to a' is found. If this final point x is known to be the point on the side of the blade at which the blade surface is parallel to the undisturbed flow in rotor coordinates, the flow conditions can be calculated. The deflection through the oblique shock originating at the blade leading edge is equal to the angle between the tangent to the leading edge and the tangent at x. The sum of the expansion 882
waves between a and x is equal in magnitude and opposite in sign to the leading-edge oblique shock.

When the direction of the undisturbed-stream velocity relative to the blades, $((90^{\circ} - \varphi)_x$, fig. 1(b)) is found, the entrance area, which in this analysis is assumed to correspond to A_e (fig. 1(c)), may now be computed as $(\pi d/n) \sin (90^{\circ} - \varphi)_x$. From figure 1(b), this area may be seen to correspond to the area of the stream tube that will enter a single passage when it is at the undisturbed entrance Mach number.

Expression for Cascade Contraction Ratio

After the entrance and minimum areas are found, the blade contraction ratio is given by

$$C_{R,max} = \frac{A_e}{A_{min}} = \frac{\frac{\pi d}{n} \sin (90^{\circ} - \varphi)_x}{\frac{\pi d}{n} \sin (90^{\circ} - \varphi)_b - t_{max}} \tag{1}$$

This expression defines a limiting blade thickness for a given entrance Mach number, $(90^{\circ} - \varphi)_x$, and $(90^{\circ} - \varphi)_b$, which agrees well with that determined from the stationary-model experiments (see section entitled EXPERIMENTAL VERIFICATION) if the model is set with the flow tangent to the correct point x.

As shown in reference 1, the actual maximum allowable diffuser contraction ratio is obtained as follows: First the mass flow per unit area is computed for the given entrance Mach number and Mach number 1 from the relation

$$\frac{\rho V}{\rho_0 a_0} = M_1 \left(1 + \frac{\gamma-1}{2} M_1^{\,2}\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \tag{2}$$

Then the isentropic contraction ratio is given as

$$C_{R,i} = \frac{(\rho V)_{M=1}}{(\rho V)_M} \tag{3}$$

The isentropic contraction ratio $C_{R,i}$ must now be multiplied by the total-pressure ratio across a normal shock at the entrance Mach number to get the actual limiting contraction ratio $C_{R,max}$. The total-pressure ratio is given by

$$\frac{P_3}{P_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{\gamma+1}{\gamma-1}} M^{\frac{2\gamma}{\gamma-1}}}{\left(\frac{2}{\gamma-1} + M^2\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma-1} M^2 - 1\right)^{\frac{1}{\gamma-1}}} \tag{4}$$

Equation (3) is now simply multiplied by equation (4) to get the limiting contraction ratio for any entrance Mach number.

EXPERIMENTAL VERIFICATION

In order to verify the contraction-ratio analysis, a two-dimensional cascade was investigated in an open-jet supersonic wind tunnel to find whether supersonic flow actually would enter the blade passage. Four contraction-ratio configurations including the computed critical $(t_{max} = 0.060)$ were used.

Apparatus and procedure. - A two-dimensional cascade of five complete blades (fig. 2(a)) was machined without twist or radial variation of gap and instrumented with static-pressure orifices. The blades were cut first with a maximum thickness t_{max} of 0.096 inch and remachined in successive steps to maximum blade thickness of 0.072, 0.060, and 0.048 inch. At each step, investigations were made at a Mach number of 1.35 to check the computed critical contraction ratio.

The entire investigation was conducted at the NACA Cleveland laboratory in a 3-inch open-jet wind tunnel designed specifically for this purpose (fig. 2(b)) with a set of nozzle blocks designed for a Mach number of 1.35. The jet was run by means of the laboratory altitude-exhaust system; room air was drawn into the 52-inch-high exhauster chamber through the nozzles. The model was located in the large chamber as shown in figure 2(b). A valve in the exhauster pipe was used to adjust the chamber pressure to equal the nozzle-exit pressure, thus eliminating either oblique shocks or expansion waves of appreciable magnitude from the downstream end of the nozzles.

Considerable caution was used in adjusting two-dimensional models to simulate exactly rotational experiments inasmuch as the upstream velocity in stator coordinates is supersonic and thereby prevents wave adjustment of the flow as in the rotating compressor.

Results and discussion. - Representative aerodynamic experimental results are given in the form of Mach number distribution (figs. 3 and 4). These Mach numbers are all based on the ratio of the observed local static pressure to the upstream total pressure; hence, the supersonic velocities given will be very nearly correct. The subsonic velocities will be high, however, as no correction is made for the total-pressure losses accompanying diffusion from supersonic to subsonic velocities. Basically, only two types of distribution exist — that in which the contraction ratio was too great for supersonic flow to enter the blade passages and that in which it was not. As this investigation was conducted for the specific purpose of checking the limiting contraction ratio, no attempt was made to evaluate the blade pressure recovery. Hence, no importance can be attached to the quantitative measurements of the pressures and Mach numbers following the passage minimum section. No provision was made for exerting back pressure on the model. For maximum efficiency, the normal shock must be located at the passage minimum section by the exertion of back pressure; therefore, no basis for performance comparison exists in the recorded data.

None of the Mach number distributions through the blade for which the contraction ratio was too great to allow supersonic flow entry into the blade passage (flow spilled) differed essentially from figure 3(a). None of the distributions for which supersonic flow did enter showed appreciable variation from figure 3(b) (flow unspilled). The Mach number distribution in figure 3(a) indicates a strong bow wave around the lower blade, probably forming a normal shock over the passage. Supersonic flow is seen to exist throughout the model in figure 3(b) with the same setting. The blade thickness of this model is reduced to 0.060 inch, however, reducing the contraction ratio to a value less than the critical computed from the proposed contraction-ratio expression.

The distributions could be altered from spilled to unspilled flow or vice versa by tilting the model in such a manner as to give an oblique shock or expansion wave at the leading edge of sufficient strength to lower or raise the entrance Mach number from the critical value. This phenomenon illustrates the necessity for alining the model at the design angle to the flow direction.

The distribution through a model whose contraction ratio was too great to allow supersonic flow entry at the free-stream Mach number M_o is shown in figure 4(a). The model, however, was tilted in the counterclockwise direction until expansion waves off the leading edge raised M_o above the critical value. Figure 4(b) shows the distribution through a model that permitted supersonic entry at the design setting, but that was caused to spill by rotating it in the clockwise direction until the leading-edge oblique shock lowered M_o below the critical value.

The difference between the C_R defined in reference 3 ($C_R = 1.092$) and that defined herein ($C_R = 1.026$) for comparable blades lies in the consideration herein of the blade curvature $\Delta(90^o - \varphi)$ between the entrance and the minimum section. This consideration promises some flexibility in choosing a practical blade thickness for relatively low supersonic Mach numbers. All or part of the total blade turning $\Delta(90^o - \varphi)$ may be included upstream of the blade-passage minimum section. The area expansion due to turning may thus be utilized to a limited extent to increase the maximum allowable blade thickness. Although contraction-ratio requirements must be met, fulfillment does not guarantee that supersonic flow will enter the passage. All changes in the entrance-region-surface slope (before the minimum section) in the direction causing compressions must still be less than the Prandtl-Meyer expansion angle that would be required to raise the flow Mach number from 1.00 to the local Mach number. Failure to observe this restriction results in local subsonic regions that result in either a strong oblique or normal shock outside the passage. The characteristic disturbances throughout the passage entrance must generally be plotted to insure that the oblique-shock reflections do not intersect to form local subsonic regions and subsequently cause spill.

SUMMARY OF RESULTS

Results of an analysis to determine the limiting contraction ratio in conjunction with a supersonic-compressor investigation indicated that the limiting contraction ratio may be closely approximated by an analytical expression involving the blade gap, the entrance flow angle, the flow angle at the blade maximum-thickness point, the number of blades, and the maximum blade thickness. The analysis further indicated that with proper regard for allowable stream turning inside the passage for a given Mach number, the turning before the minimum section may be utilized to lower the contraction ratio and consequently increase the allowable blade thickness. Characteristic lines should be constructed throughout the blade passage to make certain that the desired two-dimensional flow is obtained.

An experimental investigation of a two-dimensional model of a supersonic-compressor-blade cascade verified the analysis by allowing supersonic entry at the computed limiting contraction ratio.

Flight Propulsion Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

REFERENCES

1. Kantrowitz, Arthur, and Donaldson, Coleman du P.: Preliminary Investigation of Supersonic Diffusers. NACA ACR No. L5D20, 1945.

2. Kantrowitz, Arthur: The Supersonic Axial-Flow Compressor. NACA ACR No. L6D02, 1946.

3. Erwin, John R., Wright, Linwood C., and Kantrowitz, Arthur: Investigation of an Experimental Supersonic Axial-Flow Compressor. NACA RM No. L6J01b, 1946.

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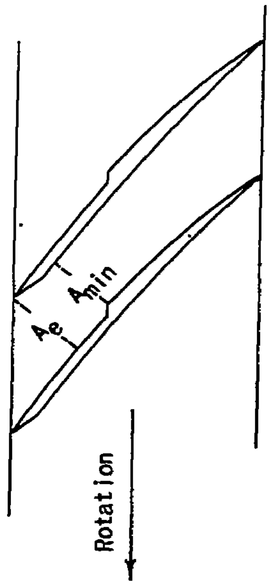
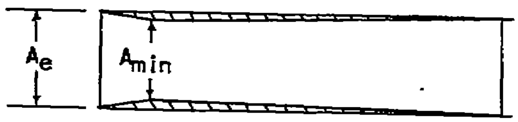
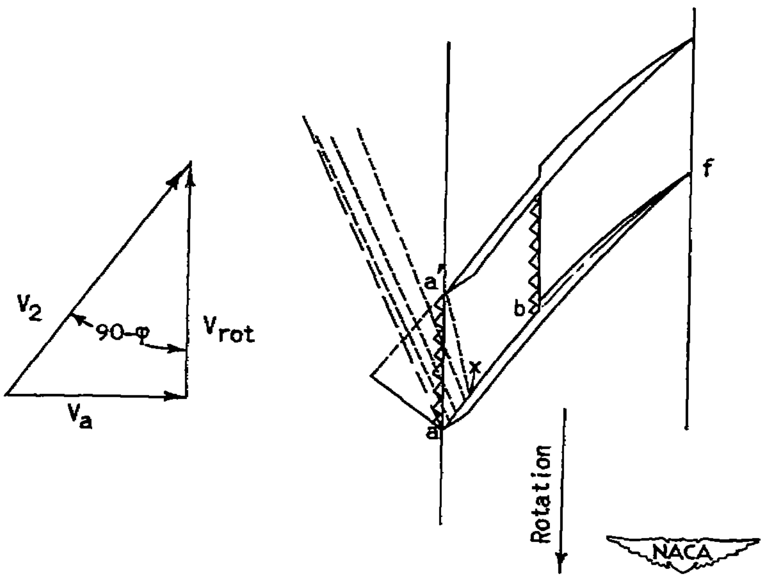


(a) Contraction ratio neglecting blade curvature (reference 3).



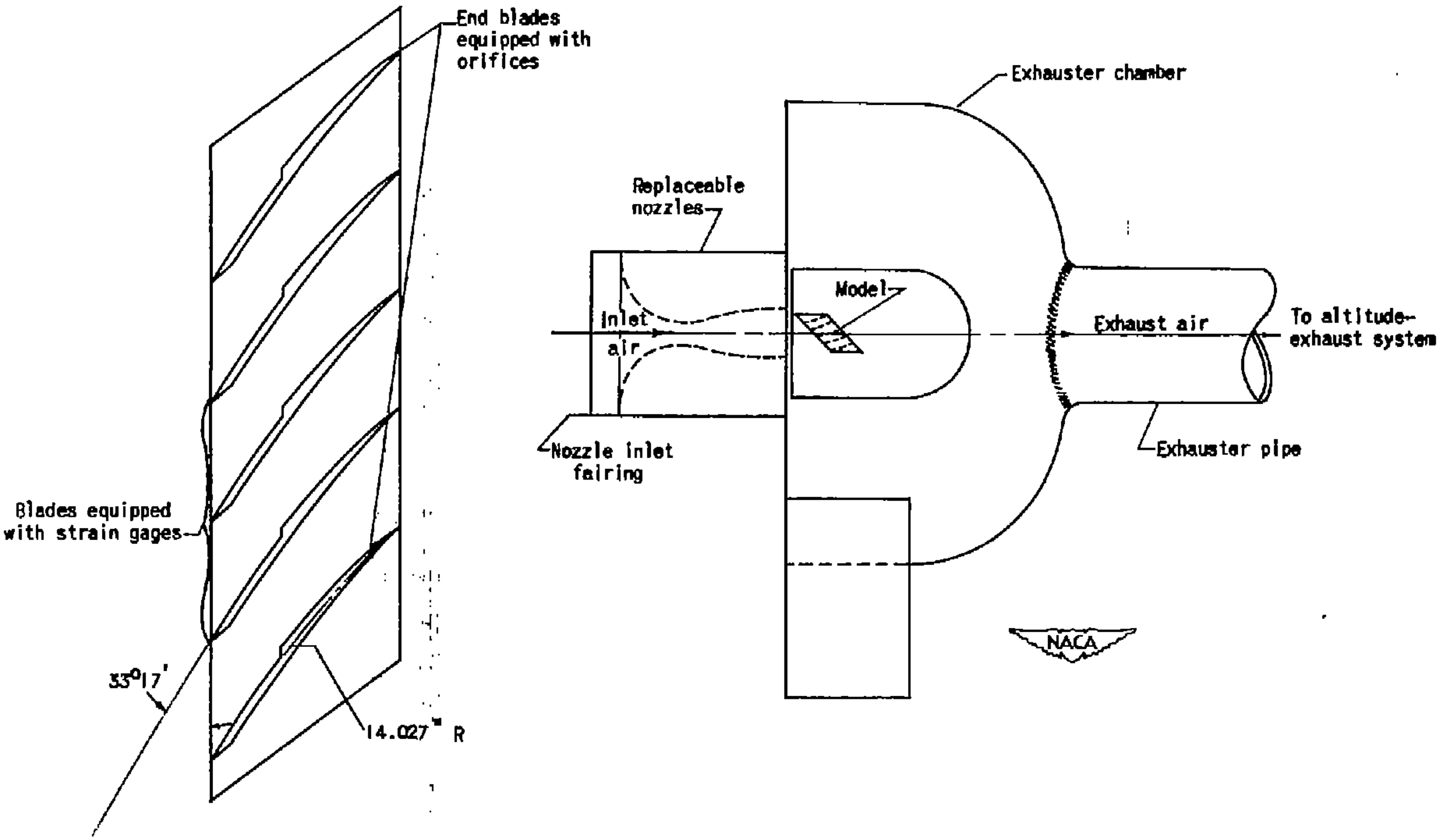
(c) Diffuser contraction ratio.



(b) Contraction ratio considering blade curvature.

Figure 1. - Contraction ratio defined ($C_R = A_e/A_{min}$).

[redacted]



(a) Supersonic-compressor-blade cascade.

(b) Supersonic 3-inch cascade tunnel.

Figure 2. - Cascade setup.

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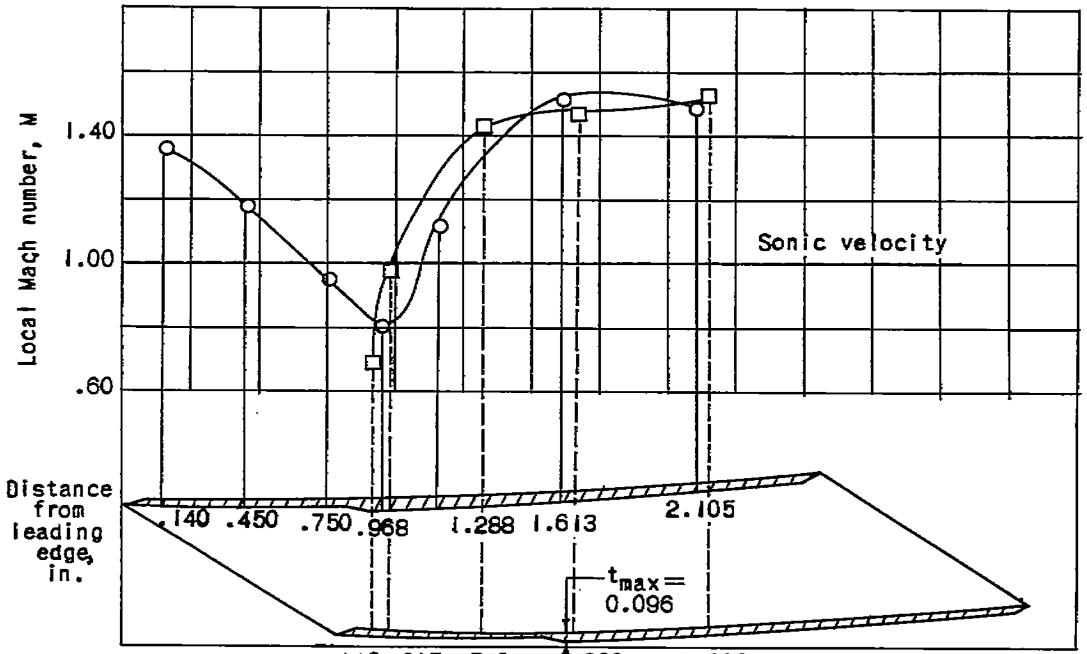
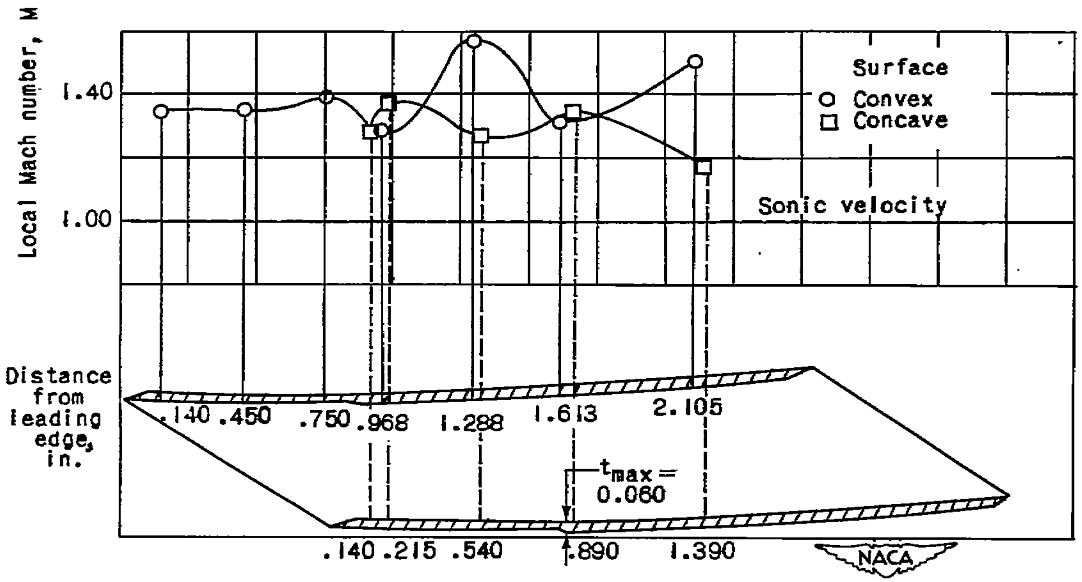


(a) Model with spilled flow at design angle with $M_o < M_{cr}$. $C_R = 1.134$; $M_{cr} = 1.662$; $M_o = 1.36$.



(b) Model with unspilled flow at design angle with $M_o > M_{cr}$. $C_R = 1.051$; $M_{cr} = 1.340$; $M_o = 1.38$.

Figure 3. - Comparison of Mach number distribution at design flow angle through supersonic-cascade model with unspilled and spilled flow. All Mach numbers based on upstream total pressure.

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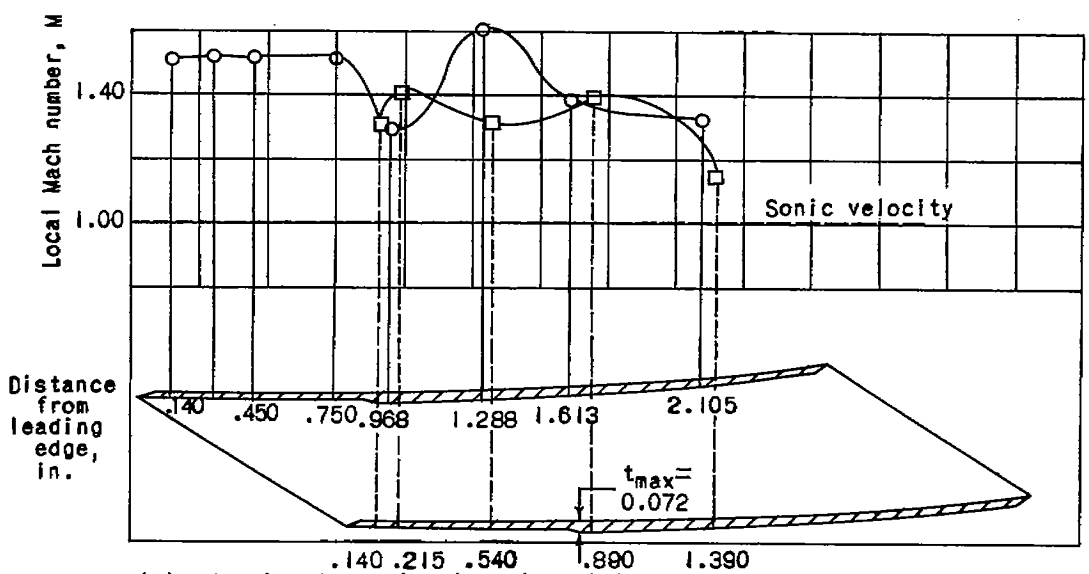
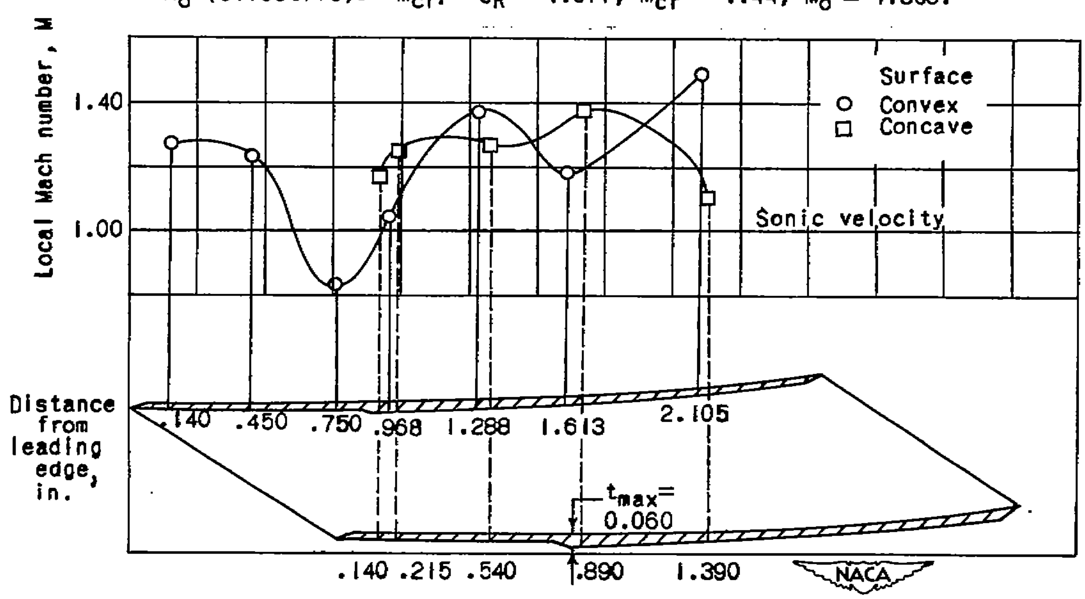


(a) Model rotated in direction giving expansion wave around leading edge of upper blade, effectively raising entrance Mach number so that M_o (effective) > M_{cr}. C_R = 1.077; M_{cr} = 1.44; M_o = 1.356.



(b) Model rotated in direction giving oblique shock off leading edge of upper blade lowering effective entrance Mach number M_o below M_{cr}. C_R = 1.051; M_{cr} = 1.340; M_o = 1.350.

Figure 4. - Comparison of Mach number distribution through two supersonic-cascade models rotated from design angle. All Mach numbers based on upstream total pressure.